Dissertation Writing

On

Graph Theory in understanding covid 19 models and the applications of tolerance graphs

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under the supervision of

Asst.Prof. Sheetal Waghmare

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Javeria Athar Salim Qureshi
K. J. Somaiya Science and Commerce
Mumbai - 400077

Contents

1 Introduction **5**

1.1 An Overview . . . 5
1.2 Definitions . . . 6
1.3 Some Important Proofs . . . 8
1.4 Thesis at a Glance . . . 10

2 Virus graph-the approach to understand Covid-19 variant? **11**

2.1 Virus graph Types . . . 11
2.2 Growth Rate . . . 13
2.3 Complexity and Limitations . . . 15

3 Tolerance graphs and their use to curb Covid-19 **16**

3.1 What are tolerance graphs? . . . 16
3.2 Its application to combat covid-19 . . . 18

4 Research Methodology **19**

4.1 Synopsis of Articles . . . 19
4.1.1 How to Write Mathematics by Halmos, P. R. . . . 19
4.1.2 How To Write Your First Paper By Steven G.Krantz . . . 21
4.2 Report on Webinars . . . 22
4.2.1 Effective Utilization Of JGATE Research Resources . . . 22
4.2.2 International Virtual Workshop On “The Craft Of Research” . 24

5 Bibliography **26**

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Certificate

K. J. Somaiya College of Science and Commerce Re-Accredited 'A' Grade by NAAC Autonomous - Affiliated to University of Mumbai

Project Completion Certificate

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Certified that the student **Javeria Athar Salim Qureshi**, Seat number: 2019189 has completed the project assigned to them and has successfully presented their work in front of a panel of examiners. They have satisfactorily defended their work.

Signature:

Date: 10-06-21

Place: Mumbai

Mentor Head of Department, Mathematics Principal

Declaration

I hereby declare that the report entitled Understanding Virus graphs and the applications of Tolerance graphs is a genuine record of the work carried out by me. Any contribution from others has been duly acknowledged. No part of this thesis has been submitted to any other university or institution for the award of any degree or diploma.

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Javeria Qureshi
2019189

Chapter 1

Introduction

1.1 An Overview

The Covid-19 pandemic has caused a lot of havoc and uncertainty in today's world.The most worrisome fact is that the structure of the virus is not stable and keeps on varying with respect to different aspects. The amount of variants has made it difficult for scientists to crack its structure with a single antivirus.The fact that there were different variants and how were the variants varying got me thinking about the possible role of graph theory in cracking out the structures. Upon further investigation,I came upon two beautiful papers on virus graphs and tolerance graphs. The former explaining about the different virus graphs covering the covid variants while the latter speaks about the applications of tolerance graphs to curb this pandemic.

The use of graph theory enables the users to understand and visualize the situations like COVID-19.Using the graph theory approach,it gets easier to understand and visualize this disease, impact and spread.

1.2 Definitions

Simple graph: A graph which is unweighted,undirected and has no loops or multiple edges.
Connected graph: In the topological sense,it is a graph in which any point is connected to any other point.
Null graph: A graph with only vertices and no edges.
Neighbour: The neighbor of the vertex v in graph H is the set of all the vertices adjacent to the vertex v in H.
Cycle graph: A simple connected graph, in which the degree of each vertex is 2, is called a cycle graph. Cn is the cycle graph on n vertices.
Star graph: A graph, in which one vertex is adjacent to n pendant vertices, is called the star graph. It is denoted by Kn.
Partite graphs: A graph H is n- partite graph if V(H) = V1 U V2 U . . . , U Vn, where all Vi are disjoint and every edge of H joins a vertex of Vi and Vj for i = j. If n = 2, 3, 4 then graphs are called Bipartite, Tripartite and four partite respectively.
Cut sets: A set of edges of a connected graph H, whose removal disconnects H, is called the disconnecting set of H. The smallest disconnecting set is called the cut set of H.
Corona product of graphs: The corona product of H and K is denoted by H $\circ$ K and obtained from a copy of H and mod V(H) copies of K, joining each vertex of H to all vertices of the graph K.
Variable sets:A set S is said to be Variable set if elements of the set S changes with respect to time or some rule. That is, the set S is not constant set. Its cardinality changes with respect to time. Sv is the notation of variable set.
Increasing Variable Set: A variable set Sv is said to be increasing variable set if $S_v(x)| < |S_v(y)I$, whenever x < y, where x and y are different times.
Decreasing Variable Set: A variable set S_v is said to be decreasing variable set if $|S_y(x)| > |S_v(y)|$, whenever $x < y$
Non Decreasing Variable Set: A variable set S_v is said to be non-decreasing variable set if $|S_v(x)| \leq| S_v(y)I$, whenever $x \leq y$
Non Increasing Variable Set: A variable set S_v is said to be non-increasing variable set if $|S_v(x)| \geq| S_v(y)I$, whenever $x \leq y$.
Stable Variable Set: A variable set S_v is said to be stable variable set if I $S_v(t)I =$ constant, for any time t. However, the set is a variable set. Elements of the set S vary according to time, but the I S_v (t) | is steady, for any time t.
Variable graph: A graph H is said to be a vertex variable graph if V (H) or E (H) is variable sets. Variable graphs are also known as V-graphs. Big network graphs are variable graph. There are two types of variable graphs.A variable graph H is said to be edge V-graph if E (H) is a variable set and V (H) is the stable variable set.A variable graph H is said to be vertex V-graph if V (H) is a variable set and E (H) is the constant variable set.
Bipartite V-graphs: A variable graph H is said to be Bipartite V-Graph if 1. V (H) = V1 U V2, where V1 and V2 are disjoint variable sets with different characteristics. 2. There exists a bond on the link or edge between vertices of V1 and vertices of V2 3. There is no any bond among the vertices of V1 only or V2 only.

N-partite V-graphs: A variable graph H is said to be n-partite V-Graph if 1. V (H) = V1 U V2 U V3,. . . .,Vn where V1, V2,V3,. . . , Vn disjoint variable sets having different characteristics. 2. There exists a bond on the link or edge between vertices of Vi Vj, for i, j and i = j.

Induced subgraph: An induced subgraph of a graph is a subgraph formed from a subset of the vertices of the graph and all of the edges connecting pairs of vertices in that subset.

Clique: A clique, C, in a graph G = (V, E) is a subset of the vertices, C V, such that every two distinct vertices are adjacent. A maximal clique is a clique that cannot be extended by including one more adjacent vertex, i.e., a maximal clique is a clique not contained in any larger clique. A maximum clique of a graph G is a clique, such that there is no clique with more vertices.

Clique number: The clique number $\omega(G)$ of a graph G is the number of vertices in a maximum clique in G.

Independence number: The maximum no. of vertices in an independent set is known as the independence number . It is denoted as $\alpha(G)$

Vertex coloring:A vertex coloring of a graph $G = (V, E)$ is a map $c : V \to S$, such that $c(v) \neq c(w)$ whenever v and w are adjacent.The smallest integer k, such that G has k -coloring, is the chromatic number that is usually denoted by $\chi(G)$. A graph G having that $\chi(G) = k$ is called k -chromatic, and if $\chi(G) \leq k$, we call Gk -colorable.

Tolerance graphs: A graph $G = (V, E)$ is a tolerance graph if each vertex $v \in V$ can be assigned a closed interval I_v and a tolerance $t_v \in \mathbb{R}^+$, such that $xy \in E$ if and only if $|I_x \cap I_y| \geq \min\{t_x, t_y\}$.

Perfect graph :→ A graph G is said to be perfect if $\chi(H)$ =$\omega(H)$ for every induced subgraph H of G,else G is said to be imperfect.

1.3 Some Important Proofs

1. For a graph G, $\chi(G) \geqslant \frac{n}{\alpha(G)}$ where n is the no. of vertices in G .

Proof: Let $\chi(G) = k \Rightarrow$ there are k colour classes. using k different colours.If s_i is a colour class then,

$$|s_i| \leqslant \alpha(G)$$

$$\sum_{i=1}^{k} |s_i| \leq \sum_{i=1}^{k} \alpha(G)$$

$$n \leqslant \alpha(G) \sum_{i=1}^{k} \cdot 1$$

$$n \leq \alpha(G) \cdot k$$

$$\Rightarrow \quad k \geqslant \frac{n}{\alpha(G)}$$

2. Every graph G with m edges satisfies $\chi(G) \leqslant \frac{1}{2} + \sqrt{2m + \frac{1}{4}}$

Proof: Let $\chi(G) = k \Rightarrow$ there are k -colour classes. Then G has atleast one edge between any two colour classes. So, choosing two colour classes from k classes at a time $= k_{c_2} = \frac{1}{2}k(k-1)$

$\Rightarrow \quad m \geqslant \frac{1}{2}k(k-1)\ k^2 - k - 2m \leq 0$
Solving for k, we get $k \leqslant \frac{1}{2} + \sqrt{2m + \frac{1}{4}}$

Thus, $\chi(G) \leq \frac{1}{2} + \sqrt{2m + 1/4}$

3. (The Four Color Theorem) Every planar graph (map) can be colored in four or fewer colors.

Proof: The planar graphs K_2, K_3, $K_4 - \{e\}$, and K_4 have respectively $1, 2, 3$ and 4 chromatic numbers. Suppose G is a planar graph with CR(G) = 5. So there are only two possibilities:
Case 1: There are five regions in G, which are adjacent to each other. Let us put one vertex in each region of G. Find geometric dual of G, say G^*. In G^*, five vertices are adjacent to each other, which forms K_5 Therefore K_5 is the subgraph of G^*. So G^* is not planar graph, which is the contradiction.
Case 2: If not all regions of G are adjacent to each other, then find geometric dual of G, say G'. In G^*, make partitions of vertices such that two vertices in G^* belong to same partition if and only if corresponding regions in G have same color. So there are five partitions only say P_1, P_2, P_3, P_4 and P_5. No vertices of the same partition are adjacent to each other. Construct new graph G1 from G * by the following

procedure.
i) Let P_1, P_2, P_3, P_4 and P_5 be the vertices of G_1 P_i and P_j are adjacent if and only if at least one vertex in Pi is adjacent to at least one vertex of P_j for $i \neq j$
ii) All Pi and Pj, for $i \neq j$ are adjacent to each other. This new graph is similar to K_5, which is the contradiction to the fact that G^* is planar graph.

4. For any graph G, the degree of its chromatic polynomial $f(G, x)$ is the number of vertices in G.

Proof: We prove it by induction on no. of vertices i.e m
Step 1: for $m = 0 \Rightarrow$ G is empty graph.
$P(G, x) = x^n$, where n is the no. of vertices.Hence verified for base step.
Step 2: Assume that the theorem holds true for graphs with m or fewer edges $\Rightarrow$ degree of the the chromatic polynomial of a graph with m or fewer edges is the number of vertices in the graph.
Step 3: Let G be a graph with ' n ' vertices & m+1 edges.
By Reduction theorem, we know that,
P(G, x)=P(G $\mid e, x) - P(G/e, x) - -(1)$
$Now, G|eisagraphwithexactlymedgesandnvertices.$
$\Rightarrow$ by Induction hypothesis, deg($P(G \mid e, x)) = n$
$\Rightarrow P(G|e, x) = a_n x^n + (termsofpowerlessthann)$
where $a_n \neq 0$ –(2)
Also, G/e has m or fewer than m edges and has exactly n-1 vertices. Again by induction, we have,
$\deg\left(P\left(G/e, x\right)\right) = n - 1$
$\Rightarrow \quad P(G/e, x) = a_{n-1}x^{n-1} +$ (terms of power less than n) $- -(3)$
$Substituting(2)\&(3)in(1), weget$P(G, x)=$a_n x^n +$ (terms of power less then n)
Hence proved.

5. A tolerance graph may not contain a chordless cycle of length greater than or equal to 5.

Proof. By the hereditary property of tolerance graphs, it is sufficient to show that $C_n$is not tolerant, for any $n \geq 5$, including n even. It is well known that its complement C'_n is not a comparability graph for any $n \geq 5, so$,C_n is not bounded tolerant. Therefore C_n is not tolerant. The same proof would show that the complements of odd length chordless cycles are not tolerant. In fact a stronger result holds; namely, a tolerance graph may not contain C'_n for $n \geq 5$, including n even. To show this it is necessary to introduce a few more concepts. We define F to be the tolerance orientation associated with a regular representation $\langle ,t \rangle$ for a tolerance graph $G = (V, E)$, by $xy \in F$ iff $xy \in E$ and $t_x < t_y$.
Clearly, a tolerance orientation is acyclic.

1.4 Thesis at a Glance

The basic intent of this thesis is to study the role of graph theory in covid-19 models.

Chapter 1 : Is introductory in nature which includes Overview of my topic,important definitions of graph theory with the help of which i will explain my thesis dissertation and some important proofs.

Chapter 2 : In this chapter,I will discuss about virus graphs and growth rates.

Chapter 3 : In this chapter,we will discuss the applications of tolerance graphs in curbing the pandemic.

Chapter 2

Virus graph-the approach to understand Covid-19 variant?

2.1 Virus graph Types

All graphs mentioned and explained are simple as well as connected.In order to understand this covid-19 theory,let us consider a bipartite variable graph V. Let V be a union of two disjoint sets V1 and V2. A vertex y in V2 is said to be an Active vertex if there exists a bond between y and any element of V1,otherwise y is said to be Passive.

To understand the variants,the author has discussed 4 types of virus graphs;namely Virus graph 1,Virus graph 2,Virus graph 3 and Virus graph 4. Let us understand the structure of these virus variants.

Some terminologies:

I: It is the variable set of vertices which is either infected by the virus or has some special properties.

N: It is the variable set of vertices which does not have the virus.

F: It is the set of vertices that can never to shifted to I or N.They work on the philosophy,"Infected once is infected forever".

S: It is the set of vertices that can never be affected by the virus.

Virus graph 1: Let H be a bipartite variable graph and V(H) be the union of I and N i.e V(H)=I U N.
Let x belong to I and y belong to N. Then if x creates a bond with y then y is shifted to I i.e the element acquires the virus. Thus N=N-{y} On the other hand,if x gets treated or loses the properties of the virus then x gets shifted to N and N=N U{x}

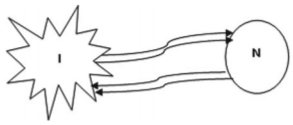


Figure 2.1: Virus graph 1

Virus graph 2: Now let H be a tripartite variable graph such that V(H) is the union of I,N and F i.e V(H)=I U N U F
Let x belong to I and y belong to N. Then if x creates a bond with y then y is shifted to I i.e the element acquires the virus. Thus N=N-{y} On the other hand,if x gets treated or loses the properties of the virus then x gets shifted to N and N=N U{x}. But if the elements i.e vertices of I are infected forever then they are moved to F. So the set is a non-decreasing variable set.

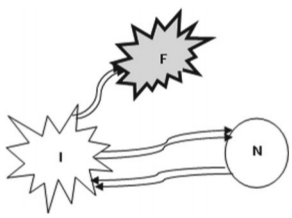


Figure 2.2: Virus graph 2

Virus graph 3: Now let H be a tripartite variable graph such that V(H) is the union of I,N and S i.e V(H)=I U N U S
Let x belong to I and y belong to N. Then if x creates a bond with y then y is shifted to I i.e the element acquires the virus. Thus N=N-{y} On the other hand,if x gets treated or loses the properties of the virus then x gets shifted to N and N=N U{x}. Vertices of this new variable set N can be shifted to S (whose vertices can never be infected by virus) if they have some special properties.
Further,we can safeguard this set S by giving it an antivirus cover and thus our set S becomes a non-decreasing variable set.
Note that the vertices of I cannot be directly transformed to S but can go to N and then get transferred to S from N.

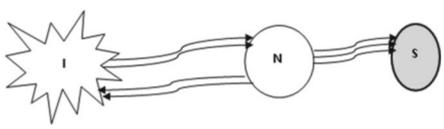


Figure 2.3: Virus graph 3

Virus graph 4: Let H be a four-partite variable graph such that V(H) is the union of I,N,F and S i.e V(H)=I U N U F U S
Let x belong to I and y belong to N. Then if x creates a bond with y then y is shifted to I i.e the element acquires the virus. Thus N=N-{y} On the other hand,if x gets

treated or loses the properties of the virus then x gets shifted to N and N=N U{x}. Vertices of this new variable set N can be shifted to S (whose vertices can never be infected by virus) if they have some special properties.
Further,we can safeguard this set S by giving it an antivirus cover and thus our set S becomes a non-decreasing variable set.
Note that the vertices of I cannot be directly transformed to S but can go to N and then get transferred to S from N.
The vertices of F can never be shifted to any other set.

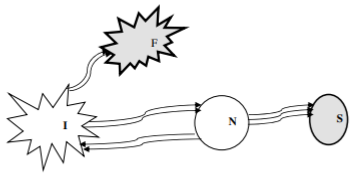


Figure 2.4: Virus graph 4

Covid-19 is a virus graph. Let C be the variable virus graph and let V(C) be the variable vertex set(set of people) and E(C) be the variable edge set. Let's start from the initial stage i.e Virus graph 1.
V(C) can be written as a union of I and N where I is the variable set of people affected by the virus and N is the set of people not affected by the virus. Now to discontinue this graph C ,we need disconnecting sets.Let D be the set of all edges adjacent to vertices of N. Note that the cut set of C will be {I} if $|I| \leq |N|, otherwise it will be$ {N}.

This is what every country was doing to control the effect of covid-19. So,either I or N is quarantined. Now let F be the set of people who did not recover and will eventually die. We now move to the stage of Virus graph 2.This can then be taken to stage of Virus graph 4 with the availability of vaccines. Virus graph 3 is excellent as it does not include set F and can be helpful for a healthy world in future.

2.2 Growth Rate

Growth rate of a virus graph is the rate of increase of active elements minus the rate of increase of passive elements.
The authors have discussed three different growth rates,One-to-One,One-to-P and One-to-All growth rates.
One-to-One growth rate is such where an active element in a variable set has 1-to-1 correspondence to an active element in another set.Here the authors have considered 2 scenarios:one where growth rate is constant and one where growth rate is not constant.
They have assumed W.L.O.G that there are 30% active elements in each set. Also assume that the number of infected people is extremely less than the total population. In this growth type, every day 30% new patients are increased in the set I. Let I_0

be the number of people infected by the virus at initial stage. Let In be the number of people will be infected after n days. As a result,

$$\mathrm{I_1 = I_0 + (0.30)I_0 = 1.3 * I_0}$$
$$\mathrm{I_2 = I_1 + (0.30)I_1 = 1.30 * I_1 = 1.3\,(1.3 * I_0) = (1.3)^2 I_0}$$

In general, $\mathrm{I_n = (1.3)^{n*} I_0}$. Let R be the rate of the virus per unit tine and $\mathrm{I_0}$ be an initial number of infected people. Therefore,

$$\mathrm{I_1 = I_0 + RI_0 = (1 + R)I_0}$$

After second time interval,

$$\mathrm{I_2 = I_1 + RI_1 = (1 + R)I_1 = (1 + R)^2 I_0}$$

After n time intervals, the number of infected people will be

$$\mathrm{I_n = (1 + R)^n I_0}$$

However,if the growth rate is not constant then After the second time interval,

$$\mathrm{I_2 = I_1 + R_2 I_1 = (1 + R_2)\,I_1 = (1 + R_2)\,(1 + R_1)\,I_0,}$$
$$\mathrm{I_3 = (1 + R_3)\,(1 + R_2)\,(1 + R_1)\,I_0}$$

After n time intervals, the number of infected people is

$$\mathrm{I_n = (1 + R_n) \ldots (1 + R_2)\,(1 + R_1)\,I_0.}$$

Moreover, at time $\mathrm{t_i}$, growth rate is $\mathrm{R_i}$ and $\mathrm{R_i}$ is repeated $\mathrm{m_i}$ times in the interval. Hence,

$$I_n = (1 + R_1)^{m_1}\,(1 + R_2)^{m_2} \ldots (1 + R_j)^{m_j}$$

where $\mathrm{1 \le j \le n}$ and $\mathrm{m_1 + m_2 + \cdots + m_j = n}$

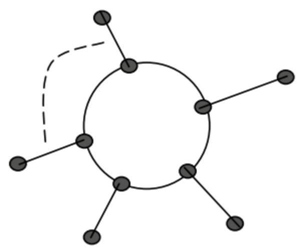


Figure 2.5: One-one growth

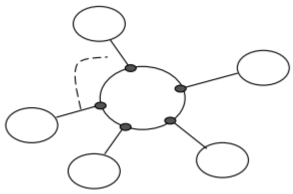


Figure 2.6: One-P growth

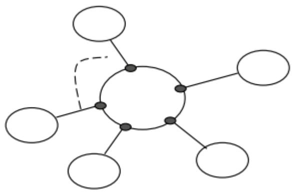


Figure 2.7: One-all growth

2.3 Complexity and Limitations

Graph theory angle makes the virus graph understandable enough and simpler. But it is still tedious to work out the cut sets. Also growth rate might be a little troublesome as it might be high.The virus graph representation in adjacency matrix is 2D including rows and columns.
On the other hand,we are constantly solving and resolving world problems with different approaches of mathematics. In this particular case,graph theory has paved the way for researchers to further investigate the virus graph mutations.
I came across the above article and was interested enough to write a short paper on it.But graph theory has no bounds. I would further like to investigate the role of Pruffer sequences in disconnecting our virus graph and also understanding if differential geometry might help to graph the rotations of the edges and vertices.

Chapter 3

Tolerance graphs and their use to curb Covid-19

3.1 What are tolerance graphs?

In graph theory, a tolerance graph is an undirected graph in which every vertex can be represented by a closed interval and a real number called its tolerance, in such a way that two vertices are adjacent in the graph whenever their intervals overlap in a length that is at least the minimum of their two tolerances. This class of graphs was introduced in 1982 by Martin Charles Golumbic and Clyde Monma, who used them to model scheduling problems in which the tasks to be modeled can share resources for limited amounts of time.

Note:Every interval graph is a tolerance graph. The complement graph of every tolerance graph is a perfectly orderable graph, from which it follows that the tolerance graphs themselves are perfect graphs.

Definition:A simple undirected graph $G = (V, E)$ on n vertices is a tolerance graph if there exists a collection $I = \{I_i \mid i = 1, 2, \ldots, n\}$ of closed intervals on the real line and a set $t = \{t_i \mid i = 1, 2, \ldots, n\}$ of positive numbers, such that for any two vertices $v_i, v_j \in V, v_iv_j \in E$ if and only if $|I_i \cap I_j| \geq \min\{t_i, t_j\}$. The pair $\langle I, t\rangle$ is called a tolerance representation of G. If G has a tolerance representation $\langle I, t\rangle$, such that $t_i \leq |I_i|$ for every $i = 1, 2, \ldots, n$, then G is called a bounded tolerance graph and $\langle I, t\rangle$ a bounded tolerance representation of G.
First, we have to remark that we are interested in a collection of intervals on the real line.
The intervals will come from various problems connected with COVID- 19 in this paper, but in general,they might arise from other applications. We will take a look in a class of graphs called intersection graphs.
Let $\mathcal{F}$ be a collection of set. The intersection graph of $\mathcal{F}$ is defined as the graph obtained by assigning a distinct vertex to each set in $\mathcal{F}$ and joining two vertices by an edge when their corresponding sets have nonempty intersection. Before defining tolerance graphs, we will introduce another family of intersection graphs, i.e., interval graphs which are actually the special case of tolerance graphs. An interval graph $G = (V, E)$ is a graph for which each vertex $v \in V$ is associated with an real interval I_v, and two vertices are connected by an edge in G if the associated

intervals have nonempty intersection.
The set of intervals $\{I_v \mid u \in V\}$ is an interval graph representation of G. In other words: $uv \in E(G) \Longleftrightarrow I_u \cap I_v \neq \emptyset, \quad$ for all $u, v \in V(G)$.

To continue further,we will first prove that tolerance graphs are perfect graphs.Perfect graphs allow polynomial-time algorithm to obtain the solution,chromatic number can be computed in polynomial time.Furthermore, finding cliques of a certain size is an NP-complete problem and determining the clique number of the graph is an NP-hard problem. However, for perfect graphs, these problems can be solved in polynomial time.

Theorem:Tolerance graphs are perfect

Proof: A graph G is perfect if for all induced subgraphs H of G, the chromatic number of H (denoted $\chi(H)$) equals the number of vertices in the largest clique in H (denoted $\omega(H)$).Also we know that a graph is tolerable only if its complement is perfect.
We begin with the background needed to show perfectly orderable graphs are perfect. Following Chvátal (1984), we define an ordered graph to be a graph $G = (V, E)$ together with a linear ordering $\prec$ on V. Label the vertex set $V = \{v_1, v_2, \ldots, v_n\}$ so that $v_i \prec v_j \Longleftrightarrow i < j$.
Consider the vertices in the order $v_1, v_2, \ldots, v_n$ and assign a value $f(v_j)$ to v_j as follows: $f(v_j)$ is the smallest positive integer not already assigned to any of v_j 's lower indexed neighbors.
The Grundy number of an ordered graph is the maximum integer
in the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$. We will denote the Grundy number of an ordered graph $(G, \prec)$ by $gr(G, \prec)$. Any linear ordering $\prec$ of $V(G)$ produces a function f which is a proper coloring of $V(G)$ using $gr(G, \prec)$ colors.
Algorithmically, the Grundy number is the number of colors used by a greedy, first fit algorithm according to the given ordering $\prec$. Thus $gr(G, \prec) \geq \chi(G)$. The inequality may be strict, although there will always be some ordering which gives equality. For example, when the path $P_4 = (V, E)$ with $V = \{a, b, c, d\}$ and $E = \{ab, bc, cd\}$ is given an ordering $\prec$ in which $a \prec b$ and $d \prec c$, then $gr\left(P_4^{\text{Ac},\gamma)} = 3$ while $\chi(P_4) = 2$. The six resulting orderings of P_4 are called obstructions in Chvátal (1984). Furthermore, Chvátal defines a linear order $\prec$ on a graph to be admissible if it has no induced obstructions and perfect if for every induced subgraph H we have $gr(H, \prec) = \chi(H)$ under the same ordering $\prec$.
If a graph has a perfect order it is called perfectly orderable. The class of co-perfectly orderable graphs consists of graphs that are the complements of perfectly orderable graphs.

3.2 Its application to combat covid-19

Looking at the current covid-19 scenario,tolarance graphs are of great help. Let G be a graph whose set of vertices correspond to the set of intervals $I = \{I_1, \ldots, I_v\}$. Each vertex is represented by $I_j = [s_j, e_j]$, where s_j and e_j are the initial time when person j is infected and the ending time of the infection, respectively. To each vertex, we add the tolerance, $t_i, i = 1, \ldots, v$ which is determined by the epidemiologists. If $|I_a \cap I_b| \leq \min\{t_a, t_b\}$, for persons a and b, then the vertices corresponding to persons a and b are connected. In the defined tolerance graph, we search for large cliques of particular sizes (or maximal cliques). Looking at a particular clique, epidemiologists could investigate if there was an event that these people were present. If such an event is found, one can conclude that events of this type are critical, and then, precaution measures could be taken to decrease the spread of the disease on similar events.

One of the real-life problems that can be modeled with tolerance graphs is connected with scheduling flights during the pandemic crisis. We are analyzing the following problem. The airport X has a rule that the passengers that are using domestic flights must be at the gates 2 h before the flight, and those getting international flights 3 h before. To decrease the possibility of spreading the disease COVID- 19 among passengers, the airport wants to schedule flights a and b for different gates if the period of time when the passengers for these two flights should be at the gates overlap for more than 30 min. Under this condition, we are searching for minimal number of different gates that can be used each day at the airport X. Let $I_j = [s_j - 2, s_j]$, where s_j is the time of the flight j scheduled for the day if the flight is domestic and let $I_j = [s_j - 3, s_j]$, where s_j is the time of the flight j scheduled for the day if the flight is international. Furthermore, let $I = \{I_1, \ldots, I_x, I_{x+1}, \ldots, I_{x+y}\}$ be the set of intervals for each flight scheduled for the exact day with x domestic and y international flights. Each interval represents a vertex of the graph G. To each vertex, we add the tolerance, $t_i, i = 1, \ldots, v$ which depend on the number of passengers that are expected at the i th flight. If $|I_a \cap I_b| \leq \min\{t_a, t_b\}$, for flights a and b, the same gates can be used. The minimum number of gates that must be used is equal to the chromatic number of the corresponding tolerance graph (each color corresponds to a gate).
This is one instance where tolerance graphs can be applied. In order to ensure safety and maintenance of protocols,one can look at tolerance graphs for a good output,especially to supervise crowd gatherings.

Chapter 4

Research Methodology

4.1 Synopsis of Articles

4.1.1 How to Write Mathematics by Halmos, P. R.

The article is in accordance to a Paul Halmos's autobiography and a film based on him: I Want To be a Mathematician. As rightly said by Paul and I quote him "It might seem unnecessary to insist that in order to say something well you must have something to say, but it's no joke. Much bad writing, mathematical and otherwise, is caused by a violation of that first principle". It's important that when you say or try to explain something, you should be very precise. You should make the reader feel like he is reading a novel irrespective of the subject.
The film follows a Question and Answer format but it can be broken down into titles:
Becoming a Mathematician
Moore method
Great teachers
Designing a course
State of education
Getting results and
writing
Relating to the book, Halmos described his approach to writing in an essay published in the book How to Write Mathematics (American Mathematical Society, 1973). One paragraph presents the essence of the process:
"The basic problem in writing mathematics is the same as in writing biology, writing a novel, or writing directions for assembling a harpsichord: the problem is to communicate an idea. To do so, and to do it clearly, you must have something to say, and you must have someone to say it to, you must organize what you want to say, and you must arrange it in the order that you want it said in, you must write it, rewrite it, and re-rewrite it several times, and you must be willing to think hard about and work hard on mechanical details such as diction, notation, and punctuation."
Halmos then expands on what he sees as the key elements of good mathematical writing.
1. Say something. To have something to say is by far the most important ingredient of good exposition.

2. Speak to someone. Ask yourself who it is that you want to reach.
3. Organize. Arrange the material so as to minimize the resistance and maximize the insight of the reader.
4. Use consistent notation. The letters (or symbols) that you use to denote the concepts that you'll discuss are worthy of thought and careful design.
5. Write in spirals. Write the first section, write the second section, rewrite the first section, rewrite the second section, write the third section, rewrite the first section, rewrite the second section, rewrite the third section, write the fourth section, and so on.
6. Watch your language. Good English style implies correct grammar, correct choice of words, correct punctuation, and common sense.
7. Be honest. Smooth the reader's way, anticipating difficulties and forestalling them. Aim for clarity, not pedantry; understanding, not fuss.
8. Remove the irrelevant. Irrelevant assumptions, incorrect emphasis, or even the absence of correct emphasis can wreak havoc.
9. Use words correctly. Think about and use with care the small words of common sense and intuitive logic, and the specifically mathematical words (technical terms) that can have a profound effect on mathematical meaning.
10.Resist symbols. The best notation is no notation; whenever it is possible to avoid the use of a complicated alphabetic apparatus, avoid it.

Halmos then concludes: "The basic problems of all expository communication are the same. . . . Content, aim, and organization, plus the vitally important details of grammar, diction, and notation—they, not showmanship, are the essential ingredients of good lectures, as well as good books." Truly, if you ever want to write an article or delve into research methodology, following Halmos's steps are a proven advantage.

4.1.2 How To Write Your First Paper By Steven G.Krantz

The writer says that the basic agenda is to understand what you are doing. To publish a paper you first need to write a paper and in order to write a paper on your work, you have to be deeply involved in it.If you wish to establish yourself in the profession and make your reputation then you must publish a research article in a peer-reviewed mathematics journal that really counts. Steven G. Krantz says that he has published more than 150 articles and knows how to do it. You cannot succeed at anything in life unless you understand what it is that you are trying to achieve.The writer also mentions his view on people stating that it gets difficult to put your work sometimes in work as nonsense. Once you understand what mathematical research is about, and how the publication process works, then you should be able to get your work into print.

The writer says that it is very important to have fire in your guts.First you should do literature survey about the research area of current interest. If you are lucky, you will have had a good and effective thesis advisor who will have given you a problem. If, this is not the case then you have to attend seminar, conferences and listen carefully to the best talks, and find out what people are thinking about.

You cannot write a good math paper by just picking up a pencil and starting to write. Some planning is needed to set the things in order and to begin by writing an outline of the paper i.e. Introduction, Background, Methodology, and Concluding remarks etc. Now you can set the length of paper to 10 or 50 pages but many journals have strict page limits, and the limit is usually about 15 or 20 pages. Some of us write directly on the computer, without working from a paper draft written by hand. If you are doing serious, deep mathematics then you will certainly have to do some of your calculations by hand.

Today journals are more demanding, and in any event you should set a higher standard for yourself. Write a paper that you yourself would want to read, if it is long and complicated make it easy for the reader. Make it accessible. Bear in mind that the referee for your paper will be a busy person who has no patience for a tract that he cannot fathom. Lay out the material so that it is rapidly apparent what your main result is, what the background for that result is, and how you are going to go about proving it. Lastly, have a nice section of Concluding Remarks, telling the reader what you have accomplished and where things might go from here?

The traditional way to submit a paper is in hard copy with all the information needed to be mentioned on cover letter. These days many journals will accept a paper electronically (via mail) with all the information included in mail and the paper should be in pdf or acrobat file format. But it takes some time for them to consider and go through,so be patient.

Finally your paper will appear either in the printed journal or the electronic journal. You should make your work available in printed or electronic media, since you are trying to establish your reputation.Its important for readers to access your work on the click of a button and save time.

4.2 Report on Webinars

4.2.1 Effective Utilization Of JGATE Research Resources

Date: 25TH DECEMBER, 2020

This webinar was conducted by Prof.Bilal Khan from KJ SOMAIYA COLLEGE. Participants and teachers were showcased the effectiveness and helpfulness of JGate Research resources during this webinar. The webinar began by warm gestures and a message from the principal.
Anyone who has attempted to write an article or a research paper will need to do a lot of reading on the subject. One of the critical issues that researchers face is access to the latest developments in their respective fields. The person will tend to look for content on the internet or even probably ransack bookshelves in a couple of libraries to source relevant and credible Content. At times like these, the researcher may face challenges in retrieving and tracking the required information from multiple sources, which hampers his/her productivity that could as well reflect in the resulting paper or article; not to mention the massive time it will require.
The webinar included topics and discussions like how to write impactful essays, how to form conclusions, how to write dissertation proposals, how to craft meaningful and colorful titles, etc.
It also included mini talks on how to write descriptive research-Descriptive Research – As the name suggests, it describes a phenomenon or a subject. Eventually, one can gather data to study a target audience or a particular subject. It does not answer questions about ‘why’ a phenomenon occurred/ occurs. Instead, it answers the question – ‘what’ are the characteristics of the phenomenon or the subject. There are three main types of Descriptive Methods – Observational methods, Case-Study methods, and Survey Methods. It also focused on Qualitative and Quantitative research and the need to avoid plagiarism.
JGate is primarily a discovery platform where one can have access to global e-journals. Apart from being the world’s most extensive database of journal articles, its unique features help save time and reduce the efforts of a researcher during the process of gathering data. Considering it houses more than 51,000,000 journal articles, it serves as a single point of access for all journals. What’s more important is that 10,000 articles are getting added daily. Its comprehensive journal classification feature comes from the fact that it has more than 47,000 indexed journals and 13,000 plus articles that are indexed based on the publishers. This makes it convenient to look up for information and thus begin with the research.
The webinar was mostly attended by students and staff of KJ Somaiya College. It was very meaningful and the knowledge was of utmost importance for students wanting to get on the path of research and research methodology.

SOMAIYA
VIDYAVIHAR
K J Somaiya College of Science & Commerce

Certificate of Participation

Qureshi Javeria Athar Salim

of KJSSC has attended the
online webinar on "**Effective Utilization of J-Gate Research Resources**"
held on 20th June, 2020 organized jointly by the
Library Development Committee and Library of
K.J.Somaiya College of Science and Commerce.

Manish Mohadikar
Librarian

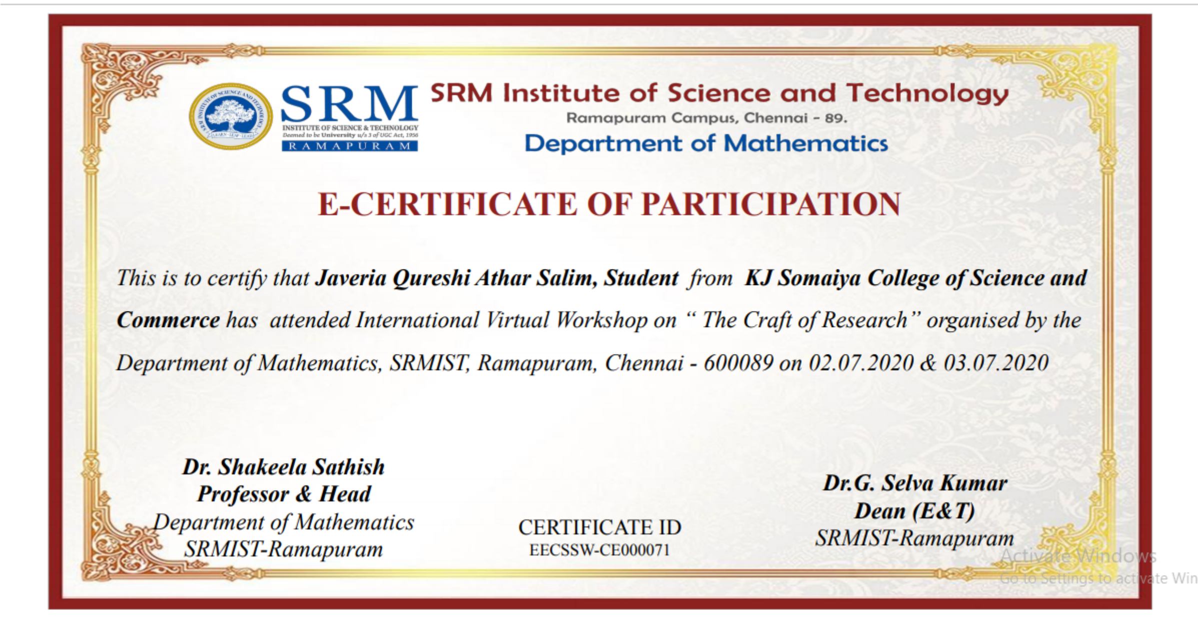
Dr. Soniya Shetty
Convenor
Library Development Committee

Dr. Pradnya Prabhu
Principal

Somaiya TRUST

4.2.2 International Virtual Workshop On "The Craft Of Research"

Date: JULY 2nd AND 3rd 2020
The International Virtual Workshop on "The Craft of Research" was organised by the Department of Mathematics, SRMIST, Ramapuram, Chennai - 600089 on 02.07.2020 03.07.2020. The session was based on how to search research articles and information to review them. It focused on how we can increase and divert to topics of our interest by emphasizing on the need to perfect and topic related keywords.
Research is getting the topmost priority in higher education today. College, an abode of Knowledge is expected to conduct genuine and high quality research, which has an impact on society. Teaching and research complement each other; they should be synthesized to bring in relevance to the academics. Management and search institutes have the responsibility to imbibe the 'Research culture' and should treat research activities as an investment rather than a cost. For undergoing high quality research and writing good research papers one requires to collect, interpret and logically document the information. The art of drawing coherent conclusions, supported by appropriate research tools and reference citation is vital for quality research work.
The session began by warm greetings from Dr. Shakeela Sathish Professor Head Department of Mathematics SRMIST-Ramapuram. Dr.Shakeela began the session on an energetic note by introducing herself, her department and their ambitions and achievements. The rest of the department also took turns to discuss the webinar's objectives. Some of the objectives were:
Provide exposure to participants on theory building and help researchers better understand concept development
Guide researchers and faculty to define research problems and draw framework based on theory building
Help faculty and research scholars to formulate research proposals
Guide participants in writing quality research papers, etc.
The session had around 148 participants (Research scholars and Faculty members) from various Campuses of SRM Institute of Science and Technology. The session concluded with a question and answer session. Research scholars and Faculty members actively involved themselves in putting forth their queries which was well addressed by the speaker.
It was a very helpful session especially for students who want to venture into the research field. I enjoyed the session and have now come to know more about research methodology.



SRM Institute of Science and Technology
Ramapuram Campus, Chennai - 89.
Department of Mathematics

E-CERTIFICATE OF PARTICIPATION

*This is to certify that **Javeria Qureshi Athar Salim, Student** from **KJ Somaiya College of Science and Commerce** has attended International Virtual Workshop on " The Craft of Research" organised by the Department of Mathematics, SRMIST, Ramapuram, Chennai - 600089 on 02.07.2020 & 03.07.2020*

Dr. Shakeela Sathish
Professor & Head
Department of Mathematics
SRMIST-Ramapuram

CERTIFICATE ID
EECSSW-CE000071

Dr.G. Selva Kumar
Dean (E&T)
SRMIST-Ramapuram

Chapter 5

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